Learn as you go: Training recurrent networks online without backtracking

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Joint work with Guillaume Charpiat and Corentin Tallec

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Algorithms to train dynamical systems

$p = \#\text{params}$

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Example: recurrent network with activation function σ,

$$h_j(t+1) = \sum_i w_{ij}\,\sigma(h_i(t)) + \sum_k r_{kj} x_k(t)$$

with $\theta = (w, r)$.

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Standard approach to compute $\frac{\partial \ell_t}{\partial \theta}$: backpropagation through time (BPTT).
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Same forward-backward structure in many problems: hidden Markov models (EM), reinforcement learning and optimal control (Bellman equations)...

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Algorithms that go forward in time must maintain the gradient of the current state with respect to the parameters:

$$G_t := \frac{\partial h(t)}{\partial \theta}$$

and then compute the gradient of the loss via the chain rule

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Sometimes, cannot even store G_t.

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- Unbiased estimate of $G_t \Longrightarrow$ unbiased estimate of the gradient of the loss function ℓ_t wrt the parameter

- The estimates are noisy but unbiased $\Longrightarrow$ over time the parameter evolves in the correct direction.

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Problem: Even if $\tilde{G}_t = \bar{v}\bar{w}^\top$ is rank-one, $\tilde{G}_{t+1}$ is full-rank again.

The rank-one trick

Proposition

Let A be a matrix and decompose A as a sum of rank-one terms,

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$$\bar{v} := \sum_i \varepsilon_i v_i \qquad \bar{w} := \sum_i \varepsilon_i w_i$$

Then $\bar{v}\bar{w}^\top$ is an unbiased, rank-one estimate of A:

$$\mathbb{E}\,\bar{v}\bar{w}^\top = A$$

Proof: expand, $\mathbb{E}\varepsilon_i^2 = 1$ and $\mathbb{E}\varepsilon_i\varepsilon_j = 0$.

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Optimal choice of λ_i: first equalize the norms of v_i and w_i.
Very important in practice.

Corollary

Apply the rank-one trick at each step. Then $\bar{v}_t \bar{w}_t^\top$ is an unbiased estimate of G_t at every step if

$$\bar{w}_{t+1} = \bar{w}_t + \sum_i \varepsilon_i \, \frac{\partial F_i}{\partial \theta}$$

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In RNNs, $\dfrac{\partial F_i}{\partial \theta}$ is sparse since $h_i(t+1)$ depends on only a small subset of parameters.

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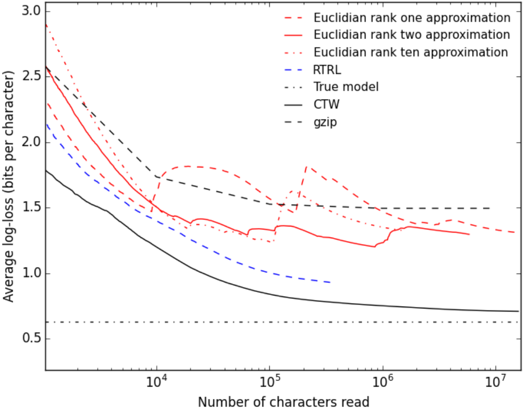
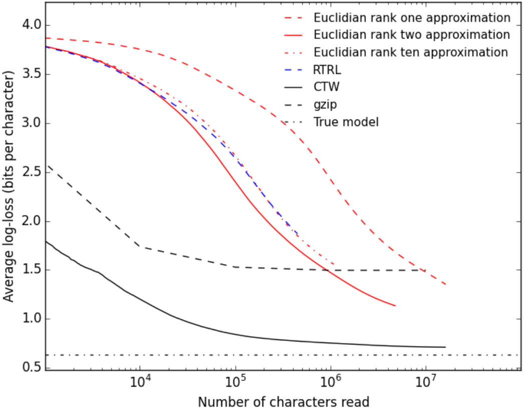
Does it work?

Large learning rate, non-Kalman: noise is clearly visible.



Compression rate (bits per characters) as a function of the number of characters read, for predicting the next character of a synthetic music notation model.

Small learning rate, non-Kalman: tracks the real gradient accurately.



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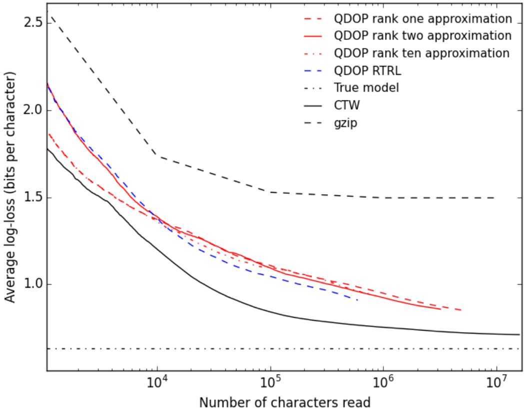

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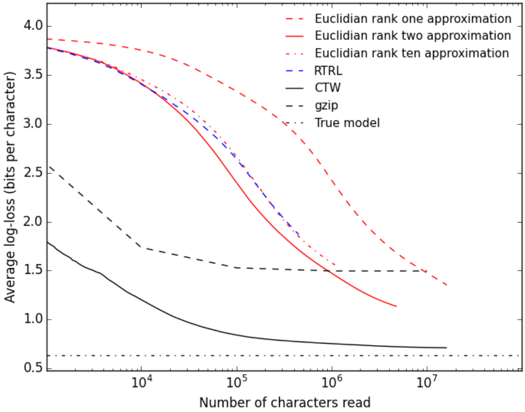

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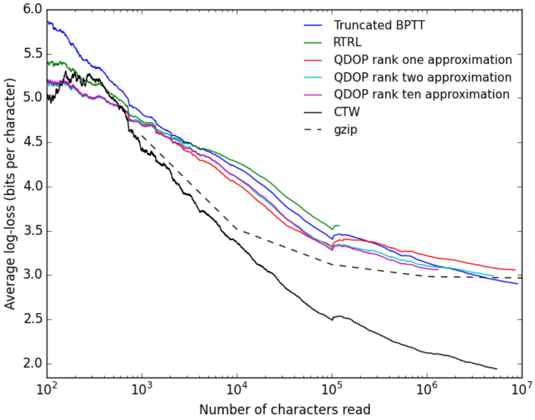
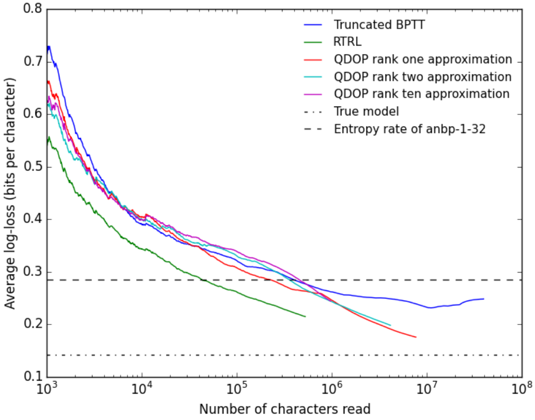
On Shakespeare's collected works, does no better, no worse than truncated backprop through time.



Compression rate (bits per characters) as a function of the number of characters read, for predicting the next character in Shakespeare's complete works.

On the $a^n b^n$ problem, clearly does better than truncated backprop through time when the span of time dependencies is longer than the truncation length for BPTT.



Compression rate (bits per characters) as a function of the number of characters read, for predicting the next character of $a^n b^n$ sequences using a leaky RNN model.

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